

D-Brane Actions with Local Kappa Symmetry¹

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Abstract

We formulate world-volume actions that describe the dynamics of Dirichlet p -branes in a flat 10d background. The fields in these theories consist of the 10d superspace coordinates (X^m, θ) and an abelian world-volume gauge field A_μ . The global symmetries are given by the N=2A or N=2B super-Poincaré group, according to whether p is even or odd. The local symmetries in the $(p + 1)$ -dimensional world volume are general coordinate invariance and a fermionic kappa symmetry.

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Introduction

During the past year (following the contribution of Polchinski [1]) the important role played by D-branes in non-perturbative superstring physics has become apparent. Many of their remarkable properties have been elucidated [2, 3, 4]. In particular, they have provided a powerful tool for the study of black holes in string theory [5]. Very recently an interesting proposal for understanding non-perturbative 11d (M theory) physics in terms of ensembles of D 0-branes has been put forward [6]. For all these reasons it is desirable to achieve as thorough an understanding of D-branes as possible. One issue that has not been explored as thoroughly for D-branes as it has been for more traditional super p -branes (without world-volume gauge fields) is the covariant formulation of the world-volume action. A crucial ingredient in such actions is a local fermionic symmetry of the world volume theory called “kappa symmetry.” It was first identified by Siegel [7] for the superparticle [8], and subsequently applied to the superstring [9]. Next it was simplified (to eliminate an unnecessary vector index) and applied to a super 3-brane in 6d [10]. Then came the super 2-brane in eleven dimensions [11], followed by all super p -branes (without world-volume gauge fields) [12].

In the case of D-branes, most studies have focused on their bosonic degrees of freedom and the coupling to bosonic background fields [13, 14]. Also, some studies have worked in a physical gauge without describing the more symmetrical gauge-invariant formulas from which they arise. Interesting as all of these studies are, there is a subtle and beautiful structure in the fermionic sector that they do not address.

The main distinction between D-branes and the previously studied super p -branes is that the field content of the world-volume theory includes an abelian vector gauge field A_μ in addition to the superspace coordinates (X^m, θ) of the ambient d -dimensional space-time. In the case of super p -branes whose only degrees of freedom are (X^m, θ) , $(p + 1)$ -dimensional actions have been formulated that have super-Poincaré symmetry in d dimensions realized as a global symmetry. In addition they have world-volume general coordinate invariance, which ensures that only the transverse components of X^m are physical, and a local fermionic kappa symmetry, which effectively eliminates half of the components of θ . This symmetry reflects the fact that the presence of the brane breaks half of the supersymmetry in d dimensions, so that half of it is realized linearly and half of it nonlinearly in the world-volume theory. The physical fermions of the world-volume theory correspond to the Goldstinos associated

to the broken supersymmetries.

The purpose of this paper is to present formulas for D-brane actions with local kappa symmetry analogous to those of the super p -branes. For this purpose the ambient space-time dimension is restricted to $d = 10$ throughout. Explicit Dirichlet p -brane actions, with all the appropriate symmetries, will be presented for all values of p ($p = 0, 1, \dots, 9$). We know from the rank of RR gauge fields that when p is even the supersymmetry should be IIA and when p is odd it should be IIB. In a physical gauge X^m gives rise to $9 - p$ degrees of freedom and A_μ gives $p - 1$ of them, for a total of 8 bosonic modes. The 32 θ coordinates are cut in half by kappa symmetry and in half again by the equation of motion, so they give rise to 8 fermionic degrees of freedom.

One case, namely $p = 2$, has been studied previously. As noted in [15], the super 2-brane action in 11d can be converted to the D 2-brane in 10d by performing a duality transformation in the world volume theory that replaces one of the X coordinates by a world-volume vector. This has been worked out in detail by Townsend [16], and some of his formulas have given us guidance in generalizing to all p . (See also [17].) One technical detail that aids the analysis is the following: We do *not* introduce an auxiliary world-volume metric field in the formulas. They have been included in most studies of super p -branes, though this was not necessary. If one attempted to incorporate them in the D-brane formulas, this would create considerable algebraic complications.

Conventions

Our conventions are the following. X^m , $m = 0, 1, \dots, 9$, denotes the 10d space-time coordinates and Γ^m are 32×32 Dirac matrices appropriate to 10d with

$$\{\Gamma^m, \Gamma^n\} = 2\eta^{mn}, \quad \text{where } \eta = (- + + \dots +). \quad (1)$$

(These Γ 's differ by a factor of i from those of ref. [18].) The Grassmann coordinates θ are space-time spinors and world-volume scalars. When p is even θ is Majorana but not Weyl. It can be decomposed as $\theta = \theta_1 + \theta_2$, where

$$\theta_1 = \frac{1}{2}(1 + \Gamma_{11})\theta, \quad \theta_2 = \frac{1}{2}(1 - \Gamma_{11})\theta. \quad (2)$$

These are Majorana–Weyl spinors of opposite chirality. When p is odd there are two Majorana–Weyl spinors θ_α ($\alpha = 1, 2$) of the same chirality. The index α will not be displayed explicitly. The group that naturally acts on it is $\text{SL}(2, \mathbb{R})$, whose generators we denote by

Pauli matrices τ_1, τ_3 . (We will mostly avoid using $i\tau_2$, which corresponds to the compact generator.) With these conventions the supersymmetry transformations (for all p) are given by

$$\delta_\epsilon \theta = \epsilon, \quad \delta_\epsilon X^m = \bar{\epsilon} \Gamma^m \theta. \quad (3)$$

World-volume coordinates are denoted σ^μ , $\mu = 0, 1, \dots, p$. The world-volume theory is supposed to have global IIA or IIB super-Poincaré symmetry. This is achieved by constructing it out of three supersymmetric quantities. Besides $\partial_\mu \theta$, they are

$$\Pi_\mu^m = \partial_\mu X^m - \bar{\theta} \Gamma^m \partial_\mu \theta, \quad (4)$$

and

$$\mathcal{F}_{\mu\nu} = F_{\mu\nu} - b_{\mu\nu}, \quad (5)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $b_{\mu\nu}$ will be defined later. Another useful quantity is the induced world-volume metric

$$G_{\mu\nu} = \eta_{mn} \Pi_\mu^m \Pi_\nu^n. \quad (6)$$

The Action

As in the case of super p -branes, the world-volume theory of a D-brane is given by a sum of two terms $S = S_1 + S_2$. The first term

$$S_1 = - \int d^{p+1} \sigma \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})} \quad (7)$$

is essentially an amalgam of the Born–Infeld and Nambu–Goto formulas. The second term

$$S_2 = \int \Omega_{p+1}, \quad (8)$$

where Ω_{p+1} is a $(p+1)$ -form, is a Wess–Zumino-type term. S_1 and S_2 are separately invariant under the global IIA or IIB super-Poincaré group as well as under $(p+1)$ -dimensional general coordinate transformations. However, local kappa symmetry will be achieved by a subtle conspiracy between them, just as in the case of super p -branes.

Under local kappa symmetry the variation $\delta\theta$ will be restricted in a way that will be determined later. In addition, we require that (whatever $\delta\theta$ is)

$$\delta X^m = \bar{\theta} \Gamma^m \delta\theta, \quad (9)$$

just as for super p -branes. It follows that

$$\delta \Pi_\mu^m = -2\delta\bar{\theta} \Gamma^m \partial_\mu \theta. \quad (10)$$

Another useful definition is the “induced γ matrix”

$$\gamma_\mu \equiv \Pi_\mu^m \Gamma_m. \quad (11)$$

Note that $\{\gamma_\mu, \gamma_\nu\} = 2G_{\mu\nu}$. These formulas imply that

$$\delta G_{\mu\nu} = -2\delta\bar{\theta}(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu)\theta. \quad (12)$$

The structure of $\mathcal{F}_{\mu\nu}$ is most easily described in terms of the 2-form $\mathcal{F} = \frac{1}{2}\mathcal{F}_{\mu\nu}d\sigma^\mu d\sigma^\nu$. Then $\mathcal{F} = F - b$, and the appropriate choice of b turns out to be [16]

$$b = -\bar{\theta}\Gamma_{11}\Gamma_m d\theta \left(dX^m + \frac{1}{2}\bar{\theta}\Gamma^m d\theta \right). \quad (13)$$

This is the formula for p even. When p is odd, Γ_{11} is replaced by τ_3 . For this choice

$$\delta_\epsilon b = -\bar{\epsilon}\Gamma_{11}\Gamma_m d\theta \left(dX^m + \frac{1}{2}\bar{\theta}\Gamma^m d\theta \right) + \frac{1}{2}\bar{\theta}\Gamma_{11}\Gamma_m d\theta \bar{\epsilon}\Gamma^m d\theta. \quad (14)$$

Then $\delta_\epsilon \mathcal{F} = 0$ if we take

$$\delta_\epsilon A = \bar{\epsilon}\Gamma_{11}\Gamma_m \theta dX^m + \frac{1}{6}(\bar{\epsilon}\Gamma_{11}\Gamma_m \theta \bar{\theta}\Gamma^m d\theta + \bar{\epsilon}\Gamma_m \theta \bar{\theta}\Gamma_{11}\Gamma^m d\theta). \quad (15)$$

The fundamental identity used to prove this, valid for any three Majorana–Weyl spinors $\lambda_1, \lambda_2, \lambda_3$ of the same chirality, is

$$\Gamma_m \lambda_1 \bar{\lambda}_2 \Gamma^m \lambda_3 + \Gamma_m \lambda_2 \bar{\lambda}_3 \Gamma^m \lambda_1 + \Gamma_m \lambda_3 \bar{\lambda}_1 \Gamma^m \lambda_2 = 0. \quad (16)$$

This formula is valid regardless of whether each of the λ ’s is an even element or an odd element of the Grassmann algebra. (Note that θ is odd and $d\theta = d\sigma^\mu \partial_\mu \theta = -\partial_\mu \theta d\sigma^\mu$ is even.) The variation of \mathcal{F} under a kappa transformation is

$$\delta \mathcal{F} = 2\delta\bar{\theta}\Gamma_{11}\Gamma_m d\theta \Pi^m, \quad (17)$$

for p even (and $\Gamma_{11} \rightarrow \tau_3$ for p odd) provided that we decree

$$\delta A = -\delta\bar{\theta}\Gamma_{11}\Gamma_m \theta \Pi^m + \frac{1}{2}\delta\bar{\theta}\Gamma_{11}\Gamma_m \theta \bar{\theta}\Gamma^m d\theta - \frac{1}{2}\delta\bar{\theta}\Gamma^m \theta \bar{\theta}\Gamma_{11}\Gamma_m d\theta. \quad (18)$$

Determination of S_2

Now let’s consider a kappa transformation of S_1 . Inserting the variations $\delta G_{\mu\nu}$ and $\delta \mathcal{F}_{\mu\nu}$ given above

$$\delta \left(-\sqrt{-\det(G + \mathcal{F})} \right) = -\frac{1}{2}\sqrt{-\det(G + \mathcal{F})} \text{tr}[(G + \mathcal{F})^{-1}(\delta G + \delta \mathcal{F})]$$

$$= 2\sqrt{-\det(G + \mathcal{F})}\delta\bar{\theta}\gamma_\mu\{(G - \mathcal{F}\Gamma_{11})^{-1}\}^{\mu\nu}\partial_\nu\theta. \quad (19)$$

For p odd Γ_{11} is replaced this time by $-\tau_3$ (since it has been moved past γ_μ). Now the key step is to rewrite this in the form

$$\delta L_1 = 2\delta\bar{\theta}\gamma^{(p)}T_{(p)}^\nu\partial_\nu\theta, \quad (20)$$

where

$$(\gamma^{(p)})^2 = 1. \quad (21)$$

It is not at all obvious that this is possible. The proof that it is, and the simultaneous determination of $\gamma^{(p)}$ and $T_{(p)}^\nu$ is the key to the whole problem. (The details of the proof will be given elsewhere [19].) Assuming that this is okay, we require that

$$\delta L_2 = 2\delta\bar{\theta}T_{(p)}^\nu\partial_\nu\theta, \quad (22)$$

so that

$$\delta(L_1 + L_2) = 2\delta\bar{\theta}(1 + \gamma^{(p)})T_{(p)}^\nu\partial_\nu\theta. \quad (23)$$

Then $\delta\bar{\theta} = \bar{\kappa}$, where $\bar{\kappa}(1 + \gamma^{(p)}) = 0$, gives the desired symmetry.

It is very useful to define

$$\rho^{(p)} = \sqrt{-\det(G + \mathcal{F})}\gamma^{(p)}, \quad (24)$$

which satisfies

$$(\rho^{(p)})^2 = -\det(G + \mathcal{F}), \quad (25)$$

and to represent it by

$$\rho^{(p)} = \frac{\epsilon^{\mu_1\mu_2\ldots\mu_{p+1}}}{(p+1)!}\rho_{\mu_1\mu_2\ldots\mu_{p+1}}, \quad (26)$$

or by a $(p+1)$ -form

$$\rho_{p+1} = \frac{\rho_{\mu_1\mu_2\ldots\mu_{p+1}}}{(p+1)!}d\sigma^{\mu_1}d\sigma^{\mu_2}\ldots d\sigma^{\mu_{p+1}}. \quad (27)$$

The requirement

$$\sqrt{-\det(G + \mathcal{F})}\gamma_\mu\{(G - \mathcal{F}\Gamma_{11})^{-1}\}^{\mu\nu} = \gamma^{(p)}T_{(p)}^\nu \quad (28)$$

can then be recast in the more convenient form

$$\rho^{(p)}\gamma_\mu = T_{(p)}^\nu(G - \mathcal{F}\Gamma_{11})_{\nu\mu}. \quad (29)$$

Writing

$$T_{(p)}^\nu = \frac{\epsilon^{\nu_1\nu_2\ldots\nu_p\nu}}{p!}T_{\nu_1\nu_2\ldots\nu_p}, \quad (30)$$

$T_{(p)}^\nu$ is characterized by the p -form

$$T_p = \frac{T_{\nu_1 \nu_2 \dots \nu_p}}{p!} d\sigma^{\nu_1} \dots d\sigma^{\nu_p}. \quad (31)$$

In this notation, the kappa variation of S_2 takes the form

$$\delta S_2 = 2(-1)^{p+1} \int \delta \bar{\theta} T_p d\theta = \delta \int \Omega_{p+1}. \quad (32)$$

It is convenient to characterize S_2 by a $(p+2)$ -form $I_{p+2} = d\Omega_{p+1}$. The preceding formula implies that

$$I_{p+2} = (-1)^{p+1} d\bar{\theta} T_p d\theta, \quad (33)$$

provided that we can show that

$$d\bar{\theta} \delta T_p d\theta + 2\delta \bar{\theta} dT_p d\theta = 0. \quad (34)$$

A corollary of this identity is the closure condition $dI_{p+2} = d\bar{\theta} dT_p d\theta = 0$.

Let us now present the solution of eqs. (25) and (29) first for the case of p even. For this purpose we define the matrix-valued one-form

$$\psi \equiv \gamma_\mu d\sigma^\mu = \Pi^m \Gamma_m, \quad (35)$$

and introduce the following formal sums of differential forms (the subscript A denotes IIA)

$$\rho_A = \sum_{p=\text{even}} \rho_{p+1} \quad \text{and} \quad T_A = \sum_{p=\text{even}} T_p. \quad (36)$$

Then the solution of eqs. (25) and (29) is described by the formulas

$$\rho_A = e^{\mathcal{F}} S_A(\psi) \quad \text{and} \quad T_A = e^{\mathcal{F}} C_A(\psi) \quad (37)$$

where

$$S_A(\psi) = \Gamma_{11} \psi + \frac{1}{3!} \psi^3 + \frac{1}{5!} \Gamma_{11} \psi^5 + \frac{1}{7!} \psi^7 + \dots \quad (38)$$

$$C_A(\psi) = \Gamma_{11} + \frac{1}{2!} \psi^2 + \frac{1}{4!} \Gamma_{11} \psi^4 + \frac{1}{6!} \psi^6 + \dots \quad (39)$$

Thus, $\rho_1 = \Gamma_{11} \psi$, $\rho_3 = \frac{1}{6} \psi^3 + \mathcal{F} \Gamma_{11} \psi$, etc., and $T_0 = \Gamma_{11}$, $T_2 = \frac{1}{2} \psi^2 + \mathcal{F} \Gamma_{11}$, etc. The fact that $T_0 \neq 0$ means that $S_2 \neq 0$ for the D 0-brane. The significant difference from the superparticle of [8] is that the D 0-brane is massive, whereas the superparticle was massless.

The proof that these expressions for ρ_A and T_A satisfy eq. (29) uses the fact that the two terms on the right-hand side of the equation correspond to $\frac{1}{2} \{\rho^{(p)}, \gamma_\mu\}$ and $\frac{1}{2} [\rho^{(p)}, \gamma_\mu]$. Using eq.(16) one can also show that this expression for T_A satisfies eq. (34).

The solution for p odd is very similar. In this case we define (the subscript B denotes IIB)

$$\rho_B = \sum_{p=\text{odd}} \rho_{p+1} \quad \text{and} \quad T_B = \sum_{p=\text{odd}} T_p. \quad (40)$$

The solution is given by

$$\rho_B = e^{\mathcal{F}} C_B(\psi) \tau_1 \quad \text{and} \quad T_B = e^{\mathcal{F}} S_B(\psi) \tau_1, \quad (41)$$

where

$$S_B(\psi) = \psi + \frac{1}{3!} \tau_3 \psi^3 + \frac{1}{5!} \psi^5 + \frac{1}{7!} \tau_3 \psi^7 + \dots \quad (42)$$

$$C_B(\psi) = \tau_3 + \frac{1}{2!} \psi^2 + \frac{1}{4!} \tau_3 \psi^4 + \frac{1}{6!} \psi^6 + \dots \quad (43)$$

Thus $\rho_2 = \frac{1}{2} \tau_1 \psi^2 + i \tau_2 \mathcal{F}$, $\rho_4 = \frac{1}{24} i \tau_2 \psi^4 + \frac{1}{2} \tau_1 \mathcal{F} \psi^2 + \frac{1}{2} i \tau_2 \mathcal{F}^2$, etc., and $T_1 = \tau_1 \psi$, $T_3 = \frac{1}{6} i \tau_2 \psi^3 + \tau_1 \mathcal{F} \psi$, etc. The quantity $\rho_0 = i \tau_2$ may be relevant to the D-instanton, which we are not considering here.

Conclusion

This brief letter has not presented proofs of three key identities — eqs. (25), (29) and (34). We plan to write a longer paper that includes the details of those proofs, as well as some additional results [19]. Other issues to explore include the extension of the formulas to non-trivial space-time backgrounds and the fixing of a physical gauge. A more challenging problem is the formulation of generalizations appropriate to the description of multiple D-branes.

As this work was nearing completion, a paper was posted that gives the kappa invariant action for the D 3-brane [20]. In fact, it also describes the coupling to background fields.

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